18. Let L be linear continuum in the order topology. Is L, connected? And what can you say about the intervals and rays in L.

19. Let X be a metrizable space. Prove that the following are equivalent:

    (a) X is compact

    (b) X is limit point compact.

    (c) X is sequentially complex.

20. State and prove Urysohn Metrization theorem.

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APRIL/MAY 2024

GMA32/DMA32 — TOPOLOGY

Time : Three hours Maximum : 75 marks

Shanmuga Industries Arts & Science College T.V.Malai

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define subspace topology.

2. Is every finite point set in a Hausdorff space closed? Justify.

3. State the Pasting lemma.

4. Show that $R \times R$ in the dictionary order topology is metrizable.

5. What is punctured Euclidean space?

6. When we will say that a space X is locally path connected?

7. Is R, the set of Reals compact? Justify.

8. Define the unit Sphere S^{n-1} and verify that it is compact or not.

9. State Tietze Extension theorem.

10. Distinguish regular and normal spaces.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Derive the criterion in terms of the bases for determining whether one topology is finer than the other.

Or

(b) Explore the relation connecting subspace topology and the order and product topologies.

12. (a) Prove that the topologies on R^n induced by the Euclidean metric d and square metric ρ are the same as the product topology on R^n.

Or

(b) Verify that the function $F : (-1,1) \to R defined by\ F(x) = \frac{x}{1-x^2}$ is a homeomorphism or not.

13. (a) Prove that a finite Cartesian product of connected spaces is connected.

Or

(b) State and prove Intermediate value theorem.

14. (a) Derive Tube Lemma.

Or

(b) State and prove the Lebesgue number lemma.

15. (a) Validate the statement: The space R_1 satisfies all the countability axioms but the second.

Or

(b) Verify that the Sorgenfrey plane R_1^2 is not normal.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive the relation connecting closure of a set and the limit points of a set. Give an example to illustrate this.

17. How does one go about constructing continuous functions from one topological space to another? Derive the rules.